Vortex formation in sheared flow driven fluctuations in nonuniform magnetized dusty gases

Arshad M. Mirza,* R. T. Faria, Jr.,† P. K. Shukla, and G. Murtaza‡
Institut für Theoretische Physik IV, Ruhr-Universität -Bochum, D-44780 Bochum, Germany
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The linear and nonlinear properties of low-frequency (in comparison with the ion gyrofrequency) electrostatic and electromagnetic waves in nonuniform, magnetized dusty plasmas with sheared plasma flows are examined. For this purpose, the multifluid dusty plasma model is employed to derive the relevant nonlinear equations for the modified dust-convective cells as well as for the coupled dust Alfvén waves and dust-convective cells. In the linear limit, we obtain dispersion relations which exhibit instabilities of both the electrostatic and electromagnetic waves. In the nonlinear case, it is shown that the newly derived dynamical equations for weakly coupled waves admit various types of vortex solutions. The results can have relevance to the understanding of the salient features of nonthermal fluctuations and coherent vortex structures in nonuniform dusty magnetoplasmas such as those in the Earth's ionosphere as well as in cometary tails and interstellar clouds. [S1063-651X(98)11101-7]

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I. INTRODUCTION

A dusty plasma is a low-temperature partially or fully ionized gas, consisting of neutral atoms, electrons, ions, and micron-sized charged particulates of solid matter (dust grains). The latter, which are extremely massive, could be either negatively or positively charged to a high degree. The electrostatic charging of the dust grains results from various processes such as the electron and ion collection from the ambient plasma, the photoelectric emission, the secondary electron and ion emission, the field emission, etc. The dust grains can acquire a huge electric charge (up to tens of thousands of an electron charge). Dusty plasmas are ubiquitous in space environments as well as in laboratory discharges [1–3].

Recently, considerable attention has been focused on studies of waves [4–8] and instabilities [9–13], as well as the nonlinear structures [13–15] in dusty plasmas. Charged dust grains can give rise to new normal modes, in addition to modifying the existing plasma wave spectra. For example, it has been found that consideration of the dust dynamics is responsible for the dust-acoustic waves [5], which are now experimentally observed [11,16]. Linear properties of electrostatic and electromagnetic waves and mechanisms for their excitation in a uniform dusty plasma were reviewed by Verheest [17].

In this paper, we study linear as well as nonlinear properties of low-frequency (in comparison with the ion gyrofrequency) electrostatic and electromagnetic waves in a nonuniform magnetized dusty gas. The latter waves contain equilibrium density and magnetic field gradients as well as sheared plasma flows. It is found that free energy stored in the latter is coupled to modified dust-convective cells and to linearly coupled dust-Alfvén waves and the dust-convective cells. On the other hand, finite amplitude electrostatic and electromagnetic waves interact nonlinearly. Accounting for convective and Lorentz force nonlinearities, we derive a set of nonlinear fluid equations in the presence of sheared plasma flows. It is found that the nonlinear equations admit new classes of coherent vortex structures.

The manuscript is organized in the following fashion. In Sec. II, we derive the relevant nonlinear equations for both the electrostatic and electromagnetic waves, by assuming that the wave frequencies are much smaller than the ion gyrofrequency. The effects of dust charge perturbations are also incorporated. However, for simplicity, we discuss in Secs. III and IV, the linear and nonlinear results, respectively, when the dust charge perturbations are ignored. Finally, Sec. V contains a brief summary and applications.

II. DERIVATION OF EQUATIONS

Let us consider a nonuniform multicomponent dusty plasma immersed in an inhomogeneous magnetic field $B_0(x)\hat{\mathbf{z}}$, where B_0 is the strength of the external magnetic field, and $\hat{\mathbf{z}}$ is the unit vector along the z axis. The dusty plasma also has equilibrium density $(\partial n_{j0}/\partial x)$ and velocity $(\partial v_{j0}/\partial x)$ gradients, which are maintained by some external sources. Here n_{j0} and v_{j0} are the equilibrium density and the magnetic field aligned flow velocity of the particle species j (j equals e for the electrons, i for the ions, and d for the dust grains).

The charge neutrality condition at equilibrium can be expressed as

$$n_{i0}=n_{e0}+Z_{d0}n_{d0}, \tag{1}$$

where Z_{d0} represents the number of charges residing on the dust grains. The dust particles are supposed to be point charges, and their sizes and the intergrain spacings are much smaller than the characteristic scale lengths (viz. gyroradii, Debye radius, etc.) of the dusty plasma. Furthermore, there

*Permanent address: Department of Physics, Quaid-i-Azam University, Islamabad 45320, Pakistan.

†Permanent address: Instituto de Fisica, Universidade Estadual de Campinas, 13083-970 Campinas, São Paolo Brazil.

‡Permanent address: Department of Physics, Quaid-i-Azam University, Islamabad 45320, Pakistan.

are sufficient numbers of dust grains in a Debye sphere, so that the collective interactions, as described below, are intact.

In the presence of low-frequency waves, the electron and ion fluid velocity perturbations in the drift approximation ($|\partial_t| \ll \omega_{ci} = eB_0/m_i c$) are

$$\mathbf{v}_e \approx \mathbf{v}_{EB} + (v_{e0} + v_{ez})\mathbf{B}_\perp / B_0 + v_{ez}\hat{\mathbf{z}} \tag{2}$$

and

$$\mathbf{v}_i \approx \mathbf{v}_{EB} + \mathbf{v}_{ip} + v_{i0}\mathbf{B}_\perp / B_0, \tag{3}$$

where $\mathbf{v}_{EB} = c\hat{\mathbf{z}} \times \nabla\phi/B_0$ and $\mathbf{v}_{ip} = -(c/B_0\omega_{ci})[\partial_t + \nu_i + \mathbf{v}_{EB} \cdot \nabla + v_{i0}\partial_z]\nabla_\perp\phi$ are the usual $\mathbf{E}\times\mathbf{B}_0$ and the ion polarization drifts, respectively; $\mathbf{E} = -\nabla\phi - (1/c)\partial_t A_z \hat{\mathbf{z}}$ is the electric field vector; $\phi(A_z)$ is the electrostatic (z component of the vector) potential and $\mathbf{B}_\perp = \nabla A_z \times \hat{\mathbf{z}}$ is the perpendicular component of the wave magnetic field, c is the speed of light, and ν_i is the ion collision frequency. The compressional magnetic field perturbation has been neglected in view of the low-β approximation. For electrostatic waves, we can set $\mathbf{B}_\perp = 0$.

The parallel component of the electron fluid velocity perturbation is determined from the z component of Ampére's law, giving

$$v_{ez} \approx (c/4\pi n_{e0} e)\nabla_\perp^2 A_z, \tag{4}$$

where $\nabla_\perp^2 = \partial_x^2 + \partial_y^2$.

The dynamics of waves in our dusty plasma system is governed by the equations of the continuity, and the momentum, which are supplemented by Poisson's equation and Ampére's law. For our purposes, for the conservation of the charge density we have

$$\partial_t(n_e - n_i) + \nu_{e,d}(n_e - n_{e0}) - \nu_{i,d}(n_i - n_{i0}) + \nabla\cdot(n_e\mathbf{v}_e - n_i\mathbf{v}_i) = 0, \tag{5}$$

Poisson's equation

$$\nabla^2\phi = 4\pi e(n_e - n_i + Z_d n_d), \tag{6}$$

and the parallel component of the electron momentum equation,

$$(\partial_t + \nu_e + v_{e0}\partial_z + \mathbf{v}_e\cdot\nabla)v_{ez} = -(e/m_e)\left[E_z + \frac{1}{c}(\mathbf{v}_e \times \mathbf{B}_\perp)\cdot\hat{\mathbf{z}}\right], \tag{7}$$

where ν_e is the electron collision frequency and the ions are assumed to be singly charged. In Eq. (5) we include [18] the source of the plasma particles which compensates for their losses due to the recombination on the surface of the dust grains. The effective collision frequencies for thermal Maxwellian plasma particle distributions are $\nu_{e,d} = \nu_{i,d}(1+P) = \nu_{ch}P(\tau+\eta)/\eta(1+\tau+\eta)$, where $P = n_{d0}Z_{d0}/n_{e0}$, $\tau = T_e/T_i$, $\eta = Z_{d0}e^2/aT_e$, and $\nu_{ch} = \omega_{pi}^2 a(1+\tau+\eta)/v_{ti}\sqrt{2\pi}$. Here a is the size of the dust grain, ω_{pi} the ion plasma frequency, v_{ti} the ion thermal velocity, and T_e (T_i) the electron (ion) temperature. For $P \ll 1$, we have $\nu_{e,d} \approx \nu_{i,d} \equiv \nu_0$.

In order to derive nonlinear set of equations, we write $n_j = n_{j0}(x) + n_{j1}$, where $n_{j1} \ll n_{j0}(x)$. Thus, substituting for the fluid velocities from Eqs. (2) and (3) into Eq. (5), we obtain, for electrostatic waves ($\mathbf{B}_\perp = \mathbf{0}$),

$$(d_t + \nu_0)\left(\nabla^2 + \frac{\omega_{pi}^2}{\omega_{ci}^2}\nabla_\perp^2\right)\phi + \frac{\omega_{pi}^2}{\omega_{ci}^2}(\nu_i + v_{i0}\partial_z)\nabla_\perp^2\phi - 4\pi ec\hat{\mathbf{z}} \times \nabla\left(\frac{Z_d n_d}{B_0}\right)\cdot\nabla\phi + 4\pi e n_{e0}\partial_z v_{ez} + 4\pi e n_{d0}(\partial_t + \mathbf{v}_{d0}\cdot\nabla)Z_{d1} = 0, \tag{8}$$

where v_{ez} is given by

$$(D_{te} + \nu_e)v_{ez} = \frac{e}{m_e}\partial_z\phi + \frac{c}{B_0}\hat{\mathbf{z}}\times\nabla v_{e0}\cdot\nabla\phi. \tag{9}$$

Here $d_t = \partial_t + \mathbf{v}_{EB}\cdot\nabla$, and $D_{te} = d_t + (v_{e0} + v_{ez})\partial_z$.

The dust charging equation for $P \ll 1$ is given by [18]

$$(\partial_t + \mathbf{v}_{d0}\cdot\nabla + \nu_{ch})q_{d1} \approx I_{e1} + I_{i1}, \tag{10}$$

where $I_{j1} = \int \sigma_j v_j f_{1j} d\mathbf{v}$ is the perturbed current in the presence of disturbances, and $\sigma_{i,e} = \pi a^2(1 \pm 2Z_d e^2/m_{i,e}v_{i,e}^2 a)$ is the charging collision cross section; the plus (minus) sign stands for the ions (electrons). The first order distribution function f_{1j} ($= f_j - F_j$) is given by

$$(\partial_t + \mathbf{v}_j\cdot\nabla)f_{1j} - \frac{q_j}{m_j c}[c\nabla\phi + \partial_t A_z\hat{\mathbf{z}} - \mathbf{v}_j \times (\mathbf{B}_0 - \hat{\mathbf{z}}\times\nabla A_z)]\cdot\nabla_{\mathbf{v}} f_j = -n_{d0}\sigma_{i,e}(v)v[f_{1j}(\mathbf{v}_j) - F_j(v_j)], \tag{11}$$

where F_j is the equilibrium velocity distribution function. It appears that the charging equation (10) is quite tedious for electromagnetic waves in nonuniform magnetized dusty plasmas.

It is straightforward to derive the relevant equations for electromagnetic waves from Eqs. (1)–(7). We have

$$(d_t + \nu_0)\left(\nabla^2 + \frac{\omega_{pi}^2}{\omega_{ci}^2}\nabla_\perp^2\right)\phi + \frac{\omega_{pi}^2}{\omega_{ci}^2}(\nu_i + v_{i0}\partial_z)\nabla_\perp^2\phi - 4\pi ec\hat{\mathbf{z}} \times \nabla\left(\frac{Z_d n_d}{B_0}\right)\cdot\nabla\phi + c d_z\nabla_\perp^2 A_z - \frac{4\pi}{B_0}\hat{\mathbf{z}}\times\nabla A_z\cdot\nabla J_{ei} + 4\pi e n_{d0}(\partial_t + \mathbf{v}_{d0}\cdot\nabla)Z_{d1} = 0, \tag{12}$$

and

$$D_{te}(1 - \lambda_e^2\nabla_\perp^2)A_z - \nu_e\lambda_e^2\nabla_\perp^2 A_z + c(\partial_z + \mathbf{S}_{v0}\cdot\nabla)\phi = 0, \tag{13}$$

where $\omega_{pi} = (4\pi n_{i0}e^2/m_i)^{1/2}$ is the ion plasma frequency, $\lambda_e = c/\ \omega_{pe}$ the collisionless electron skin depth, ω_{pe} the electron plasma frequency, $J_{\rm ei}=e(n_{i0}v_{i0}-n_{e0}v_{e0})$, and $\mathbf{S}_{v0}=\hat{\mathbf{z}}\times\nabla v_{e0}/\omega_{\rm ce}$. We have denoted $d_z\equiv\partial_z+\nabla A_z\times\hat{\mathbf{z}}/B_0$.

The origin of various terms in Eqs. (8)–(13) is now clear. The first three terms in Eqs. (8) and (12) come, respectively, from the deviation from the quasineutrality and the linear and nonlinear ion polarization drifts, whereas the fourth term in Eqs. (8) and (12) originates from the $\mathbf{E}\times\mathbf{B}$ convection of the equilibrium dust charge density, as the divergence of the difference of the electron and ion fluxes in nonuniform dusty magnetoplasmas remains finite. The fifth and sixth terms in Eq. (12) originate due to the coupling of the equilibrium plasma currents with the perturbed magnetic field; these terms do not appear in the electrostatic model (8). On the other hand, the nonlinearity on the left-hand side of Eq. (9) is due to the $\mathbf{E}\times\hat{\mathbf{z}}$ convection of the parallel (to $\hat{\mathbf{z}}$) electron velocity perturbation, whereas the important nonlinear terms in Eq. (13) come from the nonlinear parallel electron inertia as well as the nonlinear Lorentz force. The parallel phase velocity of the disturbances is assumed to be much larger than the electron thermal velocity.

III. DISPERSION RELATIONS

The local dispersion relation for electromagnetic waves is now derived by neglecting the nonlinear terms in Eqs. (12) and (13) and the dust charge fluctuations. We suppose that ϕ and A_z are proportional to $\exp(ik_yy+ik_zz-i\omega t)$, where k_y and k_z are the components of the wave vector $\mathbf{k}$, and ω is the frequency of the oscillations, and that the scale lengths of the equilibrium inhomogeneities are much larger than the wavelength. The resultant dispersion relation is of the form

$$\left[\omega-\omega_{sv}+i\nu_0-\frac{k_y^2c^2(k_zv_{i0}-i\nu_i)}{v_A^2(k^2+k_y^2c^2/v_A^2)}\right]\times\left[(\omega-k_zv_{e0})+i\frac{\nu_ek_y^2\lambda_e^2}{(1+k_y^2\lambda_e^2)}\right]=\left[k_z^2v_A^2+\frac{4\pi ck_yk_z\partial_x(J_{\rm ei}/B_0)}{(k^2+k_y^2c^2/v_A^2)}\right]\times\frac{(1+\mathbf{k}\cdot\mathbf{S}_{v0}/k_z)}{(1+k_y^2\lambda_e^2)},\tag{14}$$

where $\omega_{sv}=4\pi eck_y\partial_x(Z_{d0}n_{d0}/B_0)/(k^2+k_y^2c^2/v_A^2)$ is the dust convective cell frequency, $v_A(=c\omega_{\rm ci}/\omega_{\rm pi})$ represents the usual Alfvén velocity, and $k^2=k_y^2+k_z^2$. We note that for a highly dissipative case, the modes are damped. Also, for $k_y^2c^2/v_A^2\gg k^2$, the convective cell modes and Alfvén waves are linearly coupled. It should be noted here that the presence of ω_{sv} is attributed to the presence of the static charged dust grains, and would not arise otherwise.

In the collisionless case, the dispersion relation (14) becomes

$$\omega^2-\omega\left(\omega_{sv}+k_zv_{e0}+\frac{k_y^2c^2k_zv_{i0}}{v_A^2(k^2+k_y^2c^2/v_A^2)}\right)+k_zv_{e0}\left(\omega_{sv}+\frac{k_y^2c^2k_zv_{i0}}{v_A^2(k^2+k_y^2c^2/v_A^2)}\right)-\left(k_z^2v_A^2+\frac{4\pi k_yk_zc\partial_x(J_{\rm ei}/B_0)}{(k^2+k_y^2c^2/v_A^2)}\right)\times\frac{(1+\mathbf{k}\cdot\mathbf{S}_{v0}/k_z)}{(1+k_y^2\lambda_e^2)}=0.\tag{15}$$

Equation (15) predicts an oscillatory instability. For $\omega>k_zv_{e0}$, the threshold condition is

$$\left(k_z^2v_A^2+\frac{4\pi k_yk_zc\partial_x(J_{\rm ei}/B_0)}{(k^2+k_y^2c^2/v_A^2)}\right)\frac{(1+\mathbf{k}\cdot\mathbf{S}_{v0}/k_z)}{(1+k_y^2\lambda_e^2)}>\left(\omega_{sv}+\frac{k_y^2c^2k_zv_{i0}}{v_A^2(k^2+k_y^2c^2/v_A^2)}\right)^2.$$

On the other hand, for a highly collisional plasma in which $\nu_ek_y^2\lambda_e^2\gg(1+k_y^2\lambda_e^2)\times(\omega-k_zv_{e0})$ and $\nu_i\gg k_zv_{i0}$, we have

$$\omega=\omega_{sv}-i\left[\nu_0+\frac{k_y^2c^2\nu_i}{v_A^2(k^2+k_y^2c^2/v_A^2)}+\left\{k_z^2v_A^2+\frac{4\pi k_yk_zc\partial_x(J_{\rm ei}/B_0)}{(k^2+k_y^2c^2/v_A^2)}\right\}\frac{(1+\mathbf{k}\cdot\mathbf{S}_{v0}/k_z)}{\nu_ek_y^2\lambda_e^2}\right].\tag{16}$$

Equation (16) exhibits an instability if $\mathbf{k}\cdot\mathbf{S}_{v0}/k_z>1$ and $4\pi k_yk_zc\partial_x(J_{\rm ei}/B_0)/(k^2+k_y^2c^2/v_A^2)>k_z^2v_A^2$ as well as both the density and velocity gradients are negative. The threshold condition is

$$\left(k_z^2v_A^2+\frac{4\pi k_yk_zc\partial_x(J_{\rm ei}/B_0)}{(k^2+k_y^2c^2/v_A^2)}\right)\frac{(1+\mathbf{k}\cdot\mathbf{S}_{v0}/k_z)}{\nu_ek_y^2\lambda_e^2}>\nu_0+\frac{k_y^2c^2\nu_i}{v_A^2(k^2+k_y^2c^2/v_A^2)}.$$

Next, we focus our attention on sheared flow driven dust-convective cells. The appropriate dispersion relation for the latter can be directly derived either from Eqs. (8) and (9), or by setting $k_y^2\lambda_e^2\gg1$ in Eq. (14). We have

$$\omega^2-\omega[\omega_{sv}+k_z(v_{e0}+v_{i0})-i(\nu_0+\nu_e+\nu_i)]=\omega_{\rm LH}^2\frac{k_z^2}{k_y^2}\left[1+\frac{4\pi\partial_x(J_{\rm ei}/B_0)}{k_yk_zc}\right]\left(1+\frac{\mathbf{k}\cdot\mathbf{S}_{v0}}{k_z}\right),\tag{17}$$

where $\omega_{\rm LH}\equiv(\omega_{\rm ce}\omega_{\rm ci})^{1/2}$ is the lower-hybrid resonance frequency, and $\omega_{\rm pi}\gg\omega_{\rm ci}$ has been assumed. An inspection of Eq. (17) reveals that the dust-convective cells are driven at nonthermal level on account of free energy stored in the magnetic field aligned velocity gradient and the density inhomogeneity. The maximum growth rate of the dust-convective cell instability is roughly $\omega_{\rm LH}(k_z/k_y)|[1+4\pi\partial_x(J_{\rm ei}/B_0)/(k_y k_z c)](1+\mathbf{k}\cdot\mathbf{S}_{v0}/k_z)|^{1/2}$ for $\mathbf{S}_{v0}<0$ and $|\mathbf{k}\cdot\mathbf{S}_{v0}|>k_z$.

IV. NONLINEAR SOLUTIONS

In this section, we present the nonlinear coherent vortex solutions of Eqs. (12) and (13). Accordingly, we look for the stationary solution of the nonlinear equations (12) and (13) in the stationary frame $\xi=y+\gamma z-ut$, by assuming that $|\hat{\mathbf{z}}\times\nabla\phi\cdot\nabla|\gg(c\omega_{\rm ci}/\omega_{\rm pi}{}^2)\nabla_\perp^2 A_z\partial_z$; where γ is a constant and u is the translational speed of the vortex, by ignoring dissipative terms. For $\nabla^2\ll\omega_{\rm pi}^2\nabla_\perp^2/\omega_{\rm ci}^2$ and $\partial_z^2\ll\nabla_\perp^2$, Eqs. (12) and (13) can then be rewritten in the following forms:

$$u_{i*}\partial_\xi\nabla_\perp^2\phi-u_d\partial_\xi\phi-(c/B_0)J(\phi,\nabla_\perp^2\phi)-(v_A^2/c)\gamma\times\left[\left\{\nabla_\perp^2-\frac{4\pi}{c\gamma}\partial_x(J_{\rm ei}/B_0)\right\}\partial_\xi A_z+(1/\gamma B_0)J(A_z,\nabla_\perp^2 A_z)\right]=0 \tag{18}$$

and

$$[\partial_\xi-(c/u_{e*}B_0)(\partial_x\phi\partial_\xi-\partial_\xi\phi\partial_x)]\left[(1-\lambda_e^2\nabla_\perp^2)A_z-\frac{c\gamma_0}{u_{e*}}\phi\right]=0, \tag{19}$$

where $u_d=-4\pi ec\omega_{\rm ci}^2\partial_x(Z_{d0}n_{d0}/B_0)/\omega_{\rm pi}^2$, $J(f,g)\equiv(\partial_x f\ \partial_\xi g-\partial_x g\ \partial_\xi f)$ is the Jacobian, $\gamma_0=\gamma+(\partial_x v_{e0})/\omega_{\rm ce}$ and $u_{j*}=u-\gamma v_{j0}$.

The solutions of Eq. (19) can be obtained in two limiting cases. First, when the scale size of the nonlinear structure is much shorter than the collisionless electron skin depth, then the parallel electron inertial force can be neglected. For this case, Eq. (19) gives

$$A_z\approx(c\gamma_0/u_{e*})\phi. \tag{20}$$

Substituting for A_z from Eq. (20) into Eq. (18), we obtain

$$\partial_\xi\nabla_\perp^2\phi-\frac{c\mu}{u_{i*}B_0}J(\phi,\nabla_\perp^2\phi)-\left[\frac{u_d}{\delta u_{i*}}-\frac{4\pi\gamma_0 v_A^2}{c\,\delta u_{i*}u_{e*}}\partial_x\left(\frac{J_{\rm ei}}{B_0}\right)\right]\partial_\xi\phi=0, \tag{21}$$

where $\gamma=\gamma_0+4\pi(\lambda_e^2/c)\partial_x(J_{\rm ei}/B_0)$, $\mu=(1-\gamma_0^2 v_A^2/u_{e*}^2)/\delta$, and $\delta=(1-\gamma\gamma_0 v_A^2/u_{i*}u_{e*})$.

Equation (21) admits both the dipolar vortex [19] and the vortex street [20] solutions. The dipolar vortex appears provided that $[u_d u_{e*}-4\pi\gamma_0 v_A^2\partial_x(J_{\rm ei}/cB_0)]/(u_{i*}u_{e*}-\gamma\gamma_0 v_A^2)>0$, and that the vortex profiles are similar to those given in Ref. [19]. On the other hand, the dusty vortex street arises when $u_d=(4\pi\gamma_0 v_A^2/cu_{e*})\partial_x(J_{\rm ei}/B_0)$. Here Eq. (21) reduces to

$$\partial_\xi\nabla_\perp^2\phi-\frac{c\mu}{u_{i*}B_0}J(\phi,\nabla_\perp^2\phi)=0. \tag{22}$$

For $\mu>0$, Eq. (22) is satisfied by

$$\nabla_\perp^2\phi=\frac{4\phi_0K^2}{a_0^2}\exp\left[-\frac{2}{\phi_0}\left(\phi-\frac{u_{i*}B_0}{\mu c}x\right)\right], \tag{23}$$

where ϕ_0, K, and a_0 are some constant parameters. The solution of Eq. (23) is [20]

$$\phi=\frac{u_{i*}B_0}{\mu c}x+\phi_0\ln\left[2\cosh(Kx)+2\left(1-\frac{1}{a_0^2}\right)\cos(K\xi)\right], \tag{24}$$

which represents the Kelvin-Stuart ''cat's eyes'' that are chains of vortices for $a_0{}^2>1$.

Second, we consider the case when the scale size of the nonlinear structure is of the order of the collisionless electron skin depth and that the vortex translation speed is much larger than γv_{j0}. Here a typical bounded solution of Eq. (19) is

$$\nabla_\perp^2 A_z=\frac{1}{\lambda_e^2}\left(A_z-\frac{c\gamma_0}{u}\phi\right). \tag{25}$$

Substituting Eq. (25) into Eq. (18), we obtain

$$[\partial_\xi-(c/uB_0)(\partial_x\phi\partial_\xi-\partial_\xi\phi\partial_x)][\nabla_\perp^2\phi+\beta_1\phi-\beta_2A_z]=0. \tag{26}$$

A possible solution of Eq. (26) is

$$(\nabla_\perp^2+\beta_1-\beta_3)\phi-\beta_2A_z+\beta_3\frac{uB_0}{\mu c}x=0, \tag{27}$$

provided that $\gamma=\gamma_0+(4\pi/c)\lambda_e^2\partial_x(J_{\rm ei}/B_0)$. Here $\beta_1=-u_d/u+\gamma\gamma_0 v_A^2/u^2\lambda_e^2$, $\beta_2=-\gamma_0 v_A^2/cu\lambda_e^2$, and β_3 is an arbitrary constant of integration.

Combining Eqs. (25) and (27), we obtain

$$(\nabla_\perp^4+\chi_1\nabla_\perp^2+\chi_2)\phi-\beta_3\frac{uB_0}{\lambda_e^2\mu c}x=0, \tag{28}$$

where $\chi_1=\beta_1-\beta_3-1/\lambda_e^2$ and $\chi_2=[(\beta_3-\beta_1)+c\beta_2\gamma_0/u]/\lambda_e^2$. Equation (28) admits spatially bounded dipolar vortex solutions [21,22]. If we set $\beta_3=0$ then the solution of Eq. (28) in the outer region $(r>R)$ can be expressed as

$$\phi=[C_1K_1(s_1r)+C_2K_1(s_2r)]\cos\theta, \tag{29}$$

where C_1 and C_2 are constants, K_1 is the modified Bessel function of the first order, and $s_{1,2}^2=[-\gamma_1\pm(\gamma_1^2-4\gamma_2)^{1/2}]/2$ for $\gamma_1<0$ and $\gamma_1^2>4\gamma_2>0$. Here $\gamma_1=\beta_1-\lambda_e^{-2}$ and $\gamma_2=-(\beta_1/\lambda_e^2)+c\beta_2\gamma_0/u\lambda_e^2$. In the inner region $(r<R)$, we write the solution as

$$\phi=\left[C_3J_1(s_3r)+C_4I_1(s_4r)-\frac{\beta_3}{\lambda_e^2}\frac{uB_0}{\mu c\chi_2}r\right]\cos\theta, \quad (30)$$

where $J_1(I_1)$ is the Bessel function of the first order having real (imaginary) argument, and C_3 and C_4 are constants. Here, $s_{3,4}\equiv[(\chi_1^2-4\chi_2)^{1/2}\pm\chi_1]/2$ for $\chi_2<0$. Evidently, the outer and inner region profiles of inertial electromagnetic vortices are different from those of noninertial vortices for which the vortex scale sizes are much smaller than λ_e. It is worthwhile to mention here that the sheared equilibrium electron flow is responsible for a complete localization of the dipolar vortex in the outer region. Furthermore, the constants C_1, C_2, C_3, C_4, and β_3 can be determined by matching the inner and outer solutions of ϕ and A_z and the higher derivatives $\nabla\phi$, $\nabla_\perp^2\phi$, $\nabla_\perp A_z$, and $\nabla_\perp^2 A_z$ at the vortex interface $(r=R)$. Mikhailovskii *et al.* [22] performed this calculation, and presented explicit expressions for the various constants.

In passing, we mention that stationary solutions of Eqs. (8) and (9) can also be readily obtained in the moving frame $\xi=y+\gamma z-ut$, with $|\hat{\mathbf{z}}\times\nabla\phi\cdot\nabla|\gg(B_0/c)v_{ez}\partial_z$. In the absence of dissipative effects from Eq. (9), we have $v_{ez}=-(e\gamma_0/m_eu_{e*})\phi$, which can then be inserted into Eq. (8) to yield

$$u_{i*}\partial_\xi\nabla_\perp^2\phi-\left(u_d-\frac{\omega_{\rm LH}^2\gamma\gamma_0}{u_{e*}}\right)\partial_\xi\phi-\frac{c}{B_0}J(\phi,\nabla_\perp^2\phi)=0, \quad (31)$$

where we again assumed that $\omega_{\rm pi}^2|\nabla_\perp^2|\gg\omega_{\rm ci}^2\nabla^2$, and discarded the dust charge fluctuation. It turns out that for $u_{e*}u_{i*}-u_{e*}u_d+\gamma\gamma_0\omega_{\rm LH}^2>0$, Eq. (31) admits dipolar vortex of the type discussed in Ref. [19], whereas for $u_d=\gamma\gamma_0\omega_{\rm LH}^2/u_{e*}$, Eq. (31) reduces to the Navier-Stokes equation which is identical to Eq. (22) except that here $\mu=1$. Clearly, for this case, we will have a chain of vortices that are given by Eq. (24).

V. SUMMARY

In this paper, we investigated linear as well as nonlinear properties of coupled Alfvén and convective cells modes in nonuniform multicomponent magnetized dusty gases. For this purpose, we employed the multifluid dusty plasma model, and derived a pair of coupled nonlinear equations for both the electrostatic and electromagnetic waves in dusty plasmas that have equilibrium density, magnetic field, and magnetic-field-aligned velocity gradients. In the linear limit, we have derived local dispersion relations. The latter are analytically analyzed in order to demonstrate the existence of both the collisionless and collisional instabilities of dust-convective cells and dust-Alfvén waves. Physically, the current convective instability arises because there appears a phase lag between the parallel electron velocity perturbation and the wave potential on account of the electron velocity gradient.

Finite amplitude disturbances in nonuniform dusty gases weakly interact among themselves, and the nonlinear mode coupling can lead to the formation of dipolar vortices and vortex streets. This has been analytically shown by seeking the stationary solutions of the governing nonlinear equations (12) and (13). We find that dusty vortex streets appear in a region where the dipolar vortices are forbidden.

The present investigation of the dusty plasma wave generation and the nonlinear mode coupling leading to the formation of coherent vortex structures does not account for the dust charge fluctuations. However, it is expected that the latter should introduce a non-Landau-type dissipation [10]. Accordingly, the growth rate of the instability has to overcome the damping associated with the dust charge perturbation, whereas vortices might be amplified [20] in a dissipative dusty medium. Furthermore, the dust grains are assumed to be immobile, which is justified as long as the typical oscillation frequency is much larger than the dust plasma and dust gyrofrequencies. However, for $|\partial_t|\gg\omega_{cd}$ and $\rho_d^2|\nabla_\perp^2|\gg1$, where ω_{cd} and ρ_d are the dusty gyrofrequency and the dust gyroradius, respectively, the dust grains would follow a straight-line orbit across the external magnetic field direction. The corresponding dust number density is $n_d\approx n_{d0}\exp(Z_{d0}e\phi/T_d)$, where T_d is the dust temperature. Thus the inclusion of the dust dynamics shall give rise to a new set of nonlinear equations and new classes of unstable waves and coherent nonlinear structures whose studies are beyond the scope of the present paper. Finally, the stability of the dipolar vortices and vortex streets, as found here, has to be analyzed.

In conclusion, we stress that we have reported possible mechanisms for the generation of electrostatic as well as electromagnetic fluctuations in the presence of sheared plasma flows in nonuniform magnetized dusty gases. We have also shown that the nonlinear mode coupling provides the possibility of the formation of dusty plasma vortices which can have different scale sizes. Thus, the results of the present work should be useful in identifying the frequency and wave-number spectra of fluctuations and the features of coherent nonlinear structures which are produced by sheared plasma flows in nonuniform dusty gases. The latter are frequently found in cometary tails and interstellar clouds, as well as in many low-temperature laboratory devices.

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